



USN

Grid for USN entry

10CS42

Fourth Semester B.E. Degree Examination, Dec.2018/Jan.2019
Graph Theory and Combinatorics

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions, selecting at least TWO questions from each part.

PART - A

- 1 a. Define : (i) Complete graph (ii) Induced Subgraph (ii) Euler's circuit. Give one example for each. (05 Marks)
b. Show that there is no graph with 12 vertices and 28 edges where
i) The degree of each vertex is either 3 or 4
ii) The degree of each vertex is either 3 or 6 (05 Marks)
c. Define isomorphism of two graphs. By labeling the graphs shows that two graphs are isomorphic.

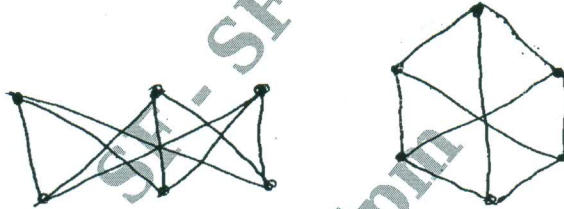


Fig Q1(c) (05 Marks)

- d. Let G = (V, E) be the undirected graph in Fig Q1(d) How many paths are there in G from a to h? How many of these paths have a length 5?

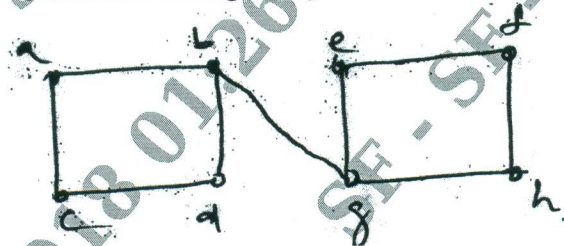


Fig Q1(d) (05 Marks)

- 2 a. A connected planar graph G with n vertices and m edges has exactly m - n + 2 regions in all of its diagrams. (07 Marks)
b. If 4 colours are used, find in how many ways can this graph be properly coloured? Hence find the chromatic number (Refer Fig Q2(b))

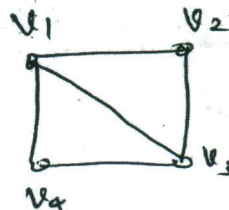


Fig Q2(b) (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- c. Consider the graph  $K_{2,3}$  shown below, Let  $\lambda$  denote the number of colours available to properly colour the vertices of this graph find
- How many proper colouring of the graph have vertices a, b coloured same
  - How many proper colourings of the graph have vertices a, b coloured differently.
  - The chromatic polynomial of the graph.

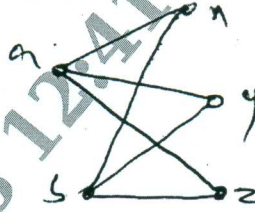


Fig Q2(c)

(06 Marks)

- 3 a. Define a binary rooted tree and show that a tree with  $n$  vertices has  $n - 1$  edge. (07 Marks)
- b. Obtain an optimal prefix code for the message LETTER RECEIVED Indirect the code. (07 Marks)
- c. Define: i) Weighted Tree ii) Prefix codes iii) Optimal prefix code. (06 Marks)
- 4 a. Explain Prim's Algorithm and find a minimal spanning tree for the weighted graph show below

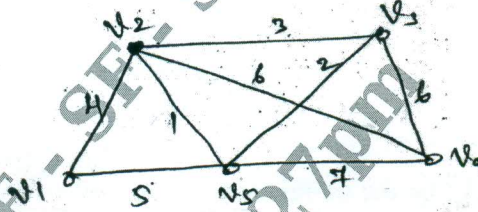


Fig Q4(a)

(06 Marks)

- b. State and prove maximum flow and minimum cut theorem. Also find the maximum flow from the vertices A and vertex Z in the network shown below

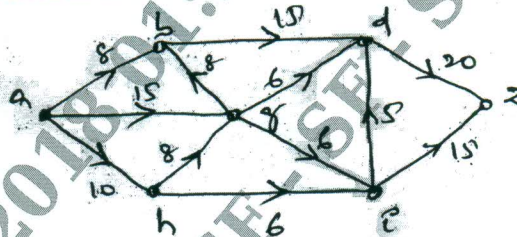


Fig Q4(b)

(07 Marks)

- c. Using the Dijkstra's algorithm, obtain the shortest path from vertex 1 to each of the other vertices in the weighted, directed network shown below indicate the weight of these shortest paths.

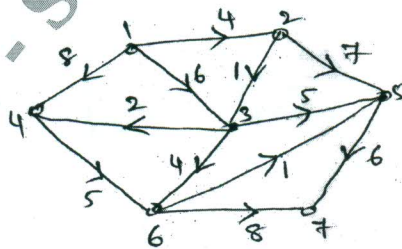


Fig Q4(c)

(07 Marks)



**PART – B**

- 5 a. How many arrangements are there for all letters in the word **SOCIOLOGICAL**? In how many of these arrangements (i) A and G are adjacent (ii) All the vowels are adjacent. (07 Marks)
- b. Determine the coefficient of  $x^2y^2z^3$  in the expansion of  $(3x - 2y - 4z)^7$  (06 Marks)
- c. Using Catalan number find the possible way of arranging four 1's and four 0's such that in each arrangement the number of 0's never exceeds the number of 1's. (07 Marks)
- 6 a. In how many ways can one distribute eight identical balls into four destined containers so that
- No container is left empty?
  - The fourth container gets an odd number of balls? (06 Marks)
- b. In how many ways can the letters in the word **CORRESPONDENTS** be arranged so that
- There is no pair of consecutive identical letters?
  - There are exactly two pairs of consecutive identical letters?
  - There are at least three pairs of consecutive identical letters? (07 Marks)
- c. An apple, a banana, a mango and an orange are to be distributed for four boys  $B_1, B_2, B_3, B_4$ . The boys  $B_1, B_2$  do not wish to have apple, the boy  $B_3$  does not want banana or mango and  $B_4$  refuses orange. In how many ways the distribution can be made so that no boy is disappointed? (07 Marks)
- 7 a. Find a generating function for each of the following sequences
- $1, 1, 0, 1, 1, 1, \dots$
  - $0, 2, 6, 12, 20, 30, 42, \dots$  (06 Marks)
- b. A bag contains a large number of red, green, white and black marbles, with at least 24 of each colour. In how many ways can one select 24 of these marbles, so that there are even numbers of white marbles and at least six black marbles? (07 Marks)
- c. A ship carries 48 flags, 12 each of the colours, red, white, blue and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal to other ships.
- How many of these signals use an even number of blue flags and an odd number of black flags?
  - How many of the signals have at least three white flags or no white flag at all? (07 Marks)
- 8 a. The number of viruses affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of viruses affected files in the system after one day. (06 Marks)
- b. If  $a_0 = 0, a_1 = 1, a_2 = 4,$  and  $a_3 = 37$  satisfy the recurrence relation  $a_{n+2} + ba_{n+1} + ca_n = 0$  for  $n \geq 0$ . Determine the constants  $b$  and  $c$  and then solve the relation for  $a_n$ . (07 Marks)
- c. Solve the recurrence relation
- $$a_{n+2} - 4a_{n+1} + 3a_n = -200, n \geq 0$$
- $$a_0 = 3000, a_1 = 3300$$
- (07 Marks)

\*\*\*\*\*